WARDALE ENGINEERING & ASSOCIATES

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CLASS 5AT 4-6-0: FUNDAMENTAL DESIGN CALCULATIONS

1. GENERAL CALCULATIONS.

1.4. TRACTIVE EFFORT DIAGRAMS.

Notes.

1. Tractive effort data is calculated for 3 specific conditions, i.e. (i) starting, (ii) maximum equivalent drawbar power and (iii) maximum design speed, 200 km/h, specifically the conditions of item [2.1.(503)] which are considered as transiently sustainable only. These three define operating extremes: the tractive effort diagrams for any other conditions may be found by the same methods as used in these calculations.
2. The SI system is mostly used. Unless otherwise stated "ton" refers to metric ton of 1000 kg.
3. Numbers in square brackets [ ] in column 2 refer to calculation item numbers in the Fundamental Design Calculations (FDC's): firstly the number identifying the calculations concerned, followed by the item number within those calculations, given in round brackets ( ), e.g. [1.1.(16)] refers to calculations 1.1. item no. (16). Where only a single number is given within square brackets, it refers to an item number within these calculations.
4. ***To save space, unit conversion factors for numerical consistency, where used, are <u>not</u> shown in the calculations. Any apparent small numerical discrepancies are due to giving data to limited places of decimals but to taking the full figure for any calculations involving that data.***
5. References are shown in superscript square brackets [ ] and are given in full at the end of the calculations.
6. Fundamental data is in **bold** type.

Item No.	Item	Unit	Amount
1	**Tractive effort diagram at starting.** By definition the inertia forces due to reciprocating masses and their balance weights are zero at starting and negligible at very low speed. Therefore their effects are entirely ignored for the starting diagram, which is based solely on the steam pressure force on the piston, with an allowance for frictional resistance. Firstly the full (forward) gear indicator diagram must be found: as the maximum tractive effort is of interest, the chosen values of contributory factors are generally those which will result in the fullest indicator diagram.		
2	At starting, boiler – steam chest pressure drop ≈	kPa	0
3	Maximum (gauge) steam chest pressure (assumed constant during cycle) ≈ [1.3.(29)] – [2]:	kPa	2 100
4	At starting, (gauge) exhaust steam pressure ≈	kPa	0
5	Piston swept volume, each end of cylinder = [1.3.(69)]:	m^3	0,122
6	Clearance volume, each end of cylinder = [6.(125)]:	m^3	0,01294
7	The 5AT peak cylinder pressure at starting is assumed = steam chest pressure = [3]:	kPa	2 100
8	For SAR 26 Class locomotive No. 3450, ΔP at point of cut-off in full forward gear at 5 - 10 km/h, as a % of the peak cylinder pressure, (deduced from starting indicator diagram[1]) ≈	%	2
9	ΔP at point of cut-off when starting and at very low speed is dependent on factors such as cylinder wall effects and the speed of valve closure, which are more optimal on the 5AT. Therefore ΔP for the 5AT is taken as:	%	1
10	Cylinder pressure at cut-off = [7] x (1 – [9]):	kPa	2 079
11	The index of expansion when starting is highly dependant on cylinder temperature: if the cylinders are 'cold' pressure will drop rapidly after valve closure due to high heat transfer to the cylinder walls, if they are fully warmed up expansion will be close to isentropic. As it is the purpose of these calculations to define the maximum tractive effort, as a check on the adequacy of the available adhesion, the latter will be taken, i.e. expansion follows the curve $pv^{1.3} = k$, where p is absolute pressure.		
12	In full forward gear, mean (front port and back port) cut-off as a % of the piston stroke, from Calculations [5] Appendix 1:	%	75,0
13	In full forward gear, mean (front port and back port) release as a % of the piston stroke, from Calculations [5] Appendix 1:	%	95,6
14	The cylinder pressure is assumed to fall in a straight line from release to exhaust pressure item [4] at starting, this being confirmed as the approximate case by most indicator diagrams of Ref. [1].		

Item No.	Item	Unit	Amount
15	In full forward gear, mean (front port and back port) start of compression as a % of the return piston stroke, from Calculations [5] Appendix 1:	%	88,7
16	The compression for fully warmed-up cylinders is effectively isentropic, see [1.3.(243)], i.e. $pv^{1.3} = k$, where p is absolute pressure.		
17	In full forward gear, mean (front port and back port) point of admission as a % of the return piston stroke, from Calculations [5] Appendix 1:	%	99,9
18	At starting, [17] can be assumed to allow peak cylinder pressure to be reached at dead centre, this being confirmed as the approximate case by most indicator diagrams of Ref. [1]and also giving the maximum tractive effort. Therefore for the 5AT cylinder pressure at dead centre = [7]:	kPa	2 100
19	The foregoing gives all data needed for drawing the estimated starting indicator diagram in full forward gear for fully warmed up cylinders, see Fig. 1.4.1. From this diagram the pressure difference across the piston is found and hence the tractive effort data tabulated below.		
20	The table columns of item [21] are as follows. (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running. (2) = connecting rod deviation, φ degrees, from the straight line joining the cylinder centre line to the driving axle centre line and found by the equation given in Fig. 2.2.3. item (1). (3) = piston displacement from f.d.c., (x) mm, by equation of item [2.1.(37)]. (4) = (90 – θ – φ), degrees. (5) = modulus of connecting rod torque arm about the driving axle centre = |[2.1.(33)] x cos(4)|, m. (6) = pressure difference across piston from Fig. 1.4.1., +ve in direction of stroke, MPa. (7) = piston thrust = (6) x mean piston area = (6) x 156 209 mm^2, kN. (8) = connecting rod thrust = (7) ÷ cos φ = (7) ÷ cos (2), kN. (9) = torque applied to main crankpin by left side connecting rod = (8) x (5), kN-m. (10) = torque on main crankpin by right side connecting rod, 90^O ahead of left rod, kN-m. (11) = combined torque on main crankpin = (9) + (10), kN-m. (12) = starting wheel rim tractive effort ≈ {0,93 x (11) ÷ (1,842 ÷ 2)} kN where 0,93 = factor to allow for frictional losses throughout the engine = [1.3.(42)] and (1,842 ÷ 2) = ([2.1.(11)] ÷ 2) = average coupled wheel radius.		

21	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	0	0	0	90	0	2,1	328,0	328,0	0	131,2	131,2	132,5
	15	1,85	15,3	73,15	0,116	2,1	328,0	328,3	38,1	122,5	160,6	162,2
	30	3,57	59,8	56,43	0,221	2,1	328,0	328,7	72,6	101,5	174,1	175,8
	45	5,06	129,7	39,94	0,307	2,1	328,0	329,3	101,1	69,7	170,8	172,5
	60	6,20	218,8	23,80	0,366	2,1	328,0	330,0	120,8	42,1	162,9	164,5
	75	6,92	319,8	8,08	0,396	2,1	328,0	330,3	130,8	6,1	136,9	138,3
	90	7,16	425,0	-7,16	0,397	2,1	328,0	330,6	131,2	0	131,2	132,5
	105	6,92	526,9	-21,92	0,371	2,1	328,0	330,3	122,5	29,9	152,4	153,9
	120	6,20	618,8	-36,20	0,323	2,0	312,4	314,3	101,5	58,5	160,0	161,6
	135	5,06	695,3	-50,06	0,257	1,73	270,2	271,3	69,7	84,6	154,3	155,8
	150	3,57	752,7	-63,57	0,178	1,51	235,9	236,4	42,1	106,6	148,7	150,2
	165	1,85	788,0	-76,85	0,091	0,43	67,2	67,3	6,1	122,5	128,6	129,9
	180	0	800	-90	0	-2,1	-328	-328	0	131,2	131,2	132,5
	195	-1,85	788,0	-103,15	0,091	2,1	328,0	328,3	29,9	130,8	160,7	162,3
	210	-3,57	752,7	-116,43	0,178	2,1	328,0	328,7	58,5	120,8	179,3	181,1
	225	-5,06	695,3	-129,94	0,257	2,1	328,0	329,3	84,6	87,1	171,7	173,4
	240	-6,20	618,8	-143,80	0,323	2,1	328,0	330,0	106,6	54,0	160,6	162,2
	255	-6,92	526,9	-158,08	0,371	2,1	328,0	330,3	122,5	10,2	132,7	134,0
	270	-7,16	425,0	-172,84	0,397	2,1	328,0	330,6	131,2	0	131,2	132,5
	285	-6,92	319,8	-188,08	0,396	2,1	328,0	330,3	130,8	38,1	168,9	170,6
	300	-6,20	218,8	-203,80	0,366	2,1	328,0	330,0	120,8	72,6	193,4	195,3
	315	-5,06	129,7	-219,94	0,307	1,81	282,7	283,8	87,1	101,1	188,2	190,1
	330	-3,57	59,8	-236,43	0,221	1,56	243,7	244,2	54,0	120,8	174,8	176,5
	345	-1,85	15,3	-253,15	0,116	0,56	87,5	87,6	10,2	130,8	141,0	142,4
	360	0	0	-270	0	-2,1	-328	-328	0	131,2	131,2	132,5

Item No.	Item	Unit	Amount
22	The starting wheel rim tractive effort data of [21](col.(12)) is graphed in Fig. 1.4.2. together with the following:		
23	Nominal design starting wheel rim tractive effort (new engine) = [1.3.(48)]:	kN	146
24	The nominal starting wheel rim tractive effort corresponding to [22] found by using piston area as in item [20](7) and coupled wheel diameter = [2.1.(11)] in the tractive effort equation item [1.3.(41)]:	kN	151,8
25	By sight, [24] is < the mean of curve [22]. This is due to the m.e.p. from the estimated indicator diagram Fig. 1.4.1. being > (coefficient item [1.3.(44)] x boiler pressure), this in turn being due to the factors influencing Fig. 1.4.1. being generally taken at that end of their range which gives maximum tractive effort, as a consequence of which curve [22] is probably a little high.		
26	Nominal adhesive weight = [1.3.(9)] x g:	kN	588,6
27	Dividing the tractive effort scale of Fig. 1.4.2. by [26] gives a scale of coefficient of adhesion, *based on nominal adhesive weight*. Relating the lines [22], [23] & [24] to this scale gives the starting coefficient of adhesion required to prevent slipping. Also given in Fig. 1.4.2. are the following:		
28	Available starting coefficient of adhesion, 0 km/h, dry rail, = [1.1.(42)(col.(3))]:	-	0,341
29	Available starting coefficient of adhesion, 0 km/h, wet rail, = [1.1.(45)(col.(3))]:	-	0,263
30	It is seen that the available starting coefficient of adhesion is greater than required on dry rail, but on wet rail it is less than required for the instantaneous tractive effort peaks (although above the values indicated for the nominal tractive efforts). This would indicate a high risk of quarter slip based on line [22]. However it is suggested in [25] that line [22] is probably on the high side, and it also applies to full forward gear and full boiler pressure acting on the pistons, conditions which will in practice seldom occur together, experience showing that trains can almost always be started with less than full boiler pressure in the steam chests and that the usual practice is to 'notch up' before opening the throttle fully.		
31	**Tractive effort diagrams at maximum equivalent drawbar power and 200 km/h.** The conditions for maximum equivalent drawbar power are defined by the indicator diagram of Fig. 1.3.1. at speed 113 km/h ([1.3.(2)]). Combining the indicator diagram of Fig. 1.3.1. with maximum design speed, 200 km/h, gives the 'overload conditions' of [2.1.(503)], which would not be sustainable for other than a short period of time. Both represent extreme operating conditions.		
32	The combined (left and right cylinder) torque on the main crankpins for the two conditions of item [31] is given in [3.(330) cols. (8) & (9)] (see [3.(329) items (8) & (9)]), and from this the respective tractive efforts are calculated in item [33]. The table columns of item [33] are as follows: (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running. (2) = combined torque on main crankpins at 113 km/h = [3.(330) col. (8)], kN-m. (3) = wheel rim tractive effort at 113 km/h ≈ {0,93 x (2) ÷ (1,842 ÷ 2)} kN, see item [20](12) for explanation. (4) = combined torque on main crankpins at 200 km/h = [3.(330) col. (9)], kN-m. (5) = wheel rim tractive effort at 200 km/h ≈ {0,93 x (4) ÷ (1,842 ÷ 2)} kN, see item [20](12) for explanation.		

33	(1)	(2)	(3)	(4)	(5)
	0	51,4	51,9	64,4	65,0
	15	67,4	68,1	70,7	71,4
	30	85,1	85,9	79,2	80,0
	45	95,9	96,8	86,7	87,5
	60	88,1	89,0	83,4	84,2
	75	60,8	61,4	65,7	66,3
	90	51,4	51,9	64,4	65,0
	105	65,6	66,2	81,0	81,8
	120	79,6	80,4	89,9	90,8
	135	85,9	86,7	85,8	86,6
	150	84,6	85,4	74,3	75,0
	165	60,4	61,0	45,0	45,4
	180	47,7	48,2	34,7	35,0
	195	63,3	63,9	58,5	59,1
	210	82,9	83,7	87,5	88,4
	225	96,1	97,0	105,3	106,3

33 (contd.)	(1)	(2)	(3)	(4)	(5)
	240	94,7	95,6	100,7	101,2
	255	65,8	66,4	62,5	63,1
	270	47,7	48,2	34,7	35,0
	285	65,1	65,7	48,2	48,7
	300	88,4	89,3	76,8	77,6
	315	106,1	107,1	106,2	107,2
	330	98,2	99,2	109,8	110,9
	345	66,2	66,8	83,2	84,0
	360	51,4	51,9	64,4	65,0

Item No.	Item	Unit	Amount
34	Available coefficient of adhesion at 113 km/h, dry rail (from Fig. 1.1.2.):	-	0,223
35	Available coefficient of adhesion at 113 km/h, wet rail (from Fig. 1.1.2.):	-	0,172
36	Available coefficient of adhesion at 200 km/h, dry rail ([1.1.(42) col.(13)]):	-	0,208
37	Available coefficient of adhesion at 200 km/h, wet rail ([1.1.(45) col.(13)]):	-	0,160
38	The wheel rim tractive effort data of [33](cols.(3) and (5)) is graphed in Fig. 1.4.2. together with the available coefficient of adhesion lines corresponding to [34] – [37]. It is seen that the available coefficient of adhesion is greater than required on dry rail at all times, but on wet rail it is less than required for the highest of the instantaneous tractive effort peaks at both the conditions of item [31]. This would indicate a high *instantaneous* slip risk factor at these conditions, in contrast to Fig. 1.1.2. which shows adequate adhesion for the *mean* tractive effort at maximum power at all speeds (note that the 200 km/h condition of item [31] is > maximum continuous power, and that the coefficient relating wheel rim to indicated output is taken more conservatively in these calculations than in calculations [1.1] (0,93 v's 0,96 ([1.1.(47)])). See also the supplementary calculations, items [39] – [79]. The following points are relevant. (1) When the variable torque from the engine is taken into account, the locomotive's power fully utilizes the available adhesion at high as well as low speeds, i.e. the design is well-balanced from this aspect. Using a larger boiler (for example as on a 4-6-2) would result in the capacity to generate more power than 60 tons of adhesive mass could transmit under all rail conditions, requiring either an increase in axle load or more coupled axles, neither of which is desirable. (2) The tractive effort at speed is highly variable (note that these variations occur many times per second and their effect is damped out by the mass of the locomotive and train to produce an apparently smooth force). The effect of this on adhesion can be beneficial. A high coefficient of adhesion is associated with a correspondingly high level of 'creep' between wheel and rail. Creep may be defined as micro-slipping: the greater it is the higher is the wheel-rail coefficient of friction until a tractive effort is reached which friction can no longer support and gross wheel slip occurs, with a coincident large reduction in coefficient of friction. With variable torque the tractive effort peaks give high creep and maximum use of adhesion, but any tendency for this to degenerate into gross wheel slip is immediately countered by an automatic reduction in tractive effort as the wheels turn towards the following trough in the tractive effort cycle, a phenomenon that can be observed at low speed as 'quarter slip'. In this way variable torque may assist to make the very most of the available adhesion. (3) The slip risk is to be reduced by complying to the fullest practical extent with the recommendations of Ref. [2]. (4) The adhesion data, calculations [1.1.(40) – (45)], appears to apply to uncontaminated rail. Contamination of the rail surface, e.g. by lubricant films, ice or frost, or wet leaves, may significantly reduce the available coefficient of adhesion. This applies to all types of locomotive. In such difficult adhesion situations steam traction has two potential advantages. Firstly, the possibility to at least partially counteract contamination by sanding ahead of the leading bogies wheels, a technique which has been found by Chapelon to improve adhesion by (a) presenting pre-sanded rails to the coupled wheels and (b) (perhaps) by the partial removal of contaminants in sand 'picked up' by the bogie wheels. To take advantage of this, the locomotive is to be equipped with a large front sandbox, sanding the bogie wheels and thus laying a ribbon of compressed sand on the rails ahead of the coupled wheels. A 'trickle' sand feed is to be available from this supply so that sanding can continue for an appreciable distance in the event of lengthy bad rail conditions being encountered. Secondly, steam traction has the potential ability to clean the rails ahead of the coupled wheels by high-pressure steam, a cleaner rail, even if wet, being preferable to a highly contaminated one.		

Item No.	Item
39	**Supplementary Calculations**. The required coefficients of adhesion shown by relating the tractive effort curves to the adhesion scale in Fig. 1.4.2. are based on the nominal adhesive weight, [26]. In practice the locomotive's adhesive weight varies continuously during coupled wheel rotation due to the dynamic augment of the driving and coupled wheels and the effects on the weight distribution of the sprung mass of the wheel rim tractive effort – drawbar pull / engine air resistance couples and the crosshead – slidebar reactions. The combined effect of these is estimated as follows, for the practical case of maximum drawbar power for forward running (at 113 km/h, item [1.3.(2)]) only.
40	**Dynamic augment of main driving axle**. This is calculated exactly by the method of FDC. 8 items [86] – [89] but for a speed of 113 km/h instead of 200 km/h, and is for forward running and *assumes the maximum possible reciprocating balance* (see [8.(146)]). Reference should be made to these items for all data not given here. The table columns of item [41] are as follows: (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running. (2) = connecting rod deviation, φ degrees, from the straight line joining the cylinder centre line to the driving axle centre line and found by the equation given in Fig. 2.2.3. item (1). (3) = piston thrust due to steam pressure (based on mean piston area), P, +ve in the direction of the stroke, = [3. (287) col.(10)] kN. (4) = Vertical component of (3) at the crankpin. Angle of inclination of cylinders = α = [2.1.(347)] = $2{,}39^{O}$. Then vertical component of (3) at the crankpin = F_{vs} = P x {sin α + tan φ cos α } kN. Sin α = 0,042, cos α = 0,999. For the various combinations of θ and P, F_{vs} is (+ve downwards): For θ ≈ (0 – 180^{O}) and +ve P, F_{vs} = |(3)| x {0,042 + (0,999 x tan |(2)|)} kN For θ ≈ (0 – 180^{O}) and –ve P, F_{vs} = – |(3)| x {0,042 + (0,999 x tan |(2)|)} kN For θ ≈ (180 – 360^{O}) and +ve P, F_{vs} = – |(3)| x {0,042 – (0,999 x tan |(2)|)} kN For θ ≈ (180 – 360^{O}) and –ve P, F_{vs} = |(3)| x {0,042 – (0,999 x tan |(2)|)} kN (5) Vertical component of the inertia force of the reciprocating masses at 113 km/h in line with the 'engine centre line' (defined above) = F_{vi1} = [3.(287)col.(7)] x sin α = 0,042 x [3.(287)col.(7)] kN (+ve downwards, i.e. sign convention of [3.(287)col.(7)] is changed). (Note [3.(287)col.(7)] is based on [3.(46)] & [2.1.(11)]). (6) Vertical force on the main crankpin due to the unbalanced component of the inertia load at the crankpin at 90^{O} to the engine centre line, F_{vi2}, is given by cos α x equation (6) of item [8.(86)] with n = coupled wheel rotational speed at 113 km/h = 5,42 Hz with wheel o/d = [2.1.(11)], giving F_{vi2} = –1,098 x ((6,46 x sin 2θ) + (sin θ x cos 2θ)) kN = [41] col (6) (+ve downwards). (7) Net vertical load on crankpin of left main driving wheel = (4) + (5) + (6) kN, (+ve downwards). (8) Net vertical load on crankpin of right main driving wheel = (7) advanced by 90^{O}, kN (+ve downwards). (9) R_l = wheel – rail reaction at left main driving wheel due to (7) and (8) is given by: R_l = {(7) x (([3.(5)] ÷ 2) + [8.(69)]) – (8) x (([3.(5)] ÷ 2) – [8.(69)])} ÷ [8.(68)] kN (+ve downwards). (10) R_r = wheel – rail reaction at right main driving wheel due to (7) and (8) is given by: R_r = {(8) x (([3.(5)] ÷ 2) + [8.(69)]) – (7) x (([3.(5)] ÷ 2) – [8.(69)])} ÷ [8.(68)] kN (+ve downwards).

Item 41:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	0	234,3	9,84	-5,77	0	4,07	20,72	1,01	23,78
15	1,85	301,5	22,39	-5,50	-3,79	13,10	19,87	11,85	21,12
30	3,57	303,8	31,69	-4,74	-6,42	20,53	19,03	20,81	18,75
45	5,06	299,1	39,02	-3,62	-7,09	28,31	14,78	30,80	12,29
60	6,20	254,6	38,32	-2,24	-5,67	30,41	7,94	34,54	3,81
75	6,92	166,4	27,16	-0,77	-2,63	23,76	1,41	27,87	-2,70
90	7,16	113,3	18,98	0,64	1,10	20,72	-5,38	25,52	-10,18
105	6,92	82,8	13,52	1,88	4,47	19,87	-1,85	23,87	-5,85
120	6,20	63,3	9,53	2,88	6,62	19,03	4,41	21,72	1,72
135	5,06	31,2	4,07	3,62	7,09	14,78	10,54	15,56	9,76
150	3,57	-19,5	-2,03	4,10	5,87	7,94	15,28	6,59	16,63
165	1,85	-84,4	-6,27	4,38	3,30	1,41	13,38	-0,79	15,58
180	0	-234,3	-9,84	4,46	0	-5,38	10,75	-8,35	13,72
195	-1,85	301,5	-2,93	4,38	-3,30	-1,85	9,41	-3,92	11,48
210	-3,57	303,8	6,18	4,10	-5,87	4,41	8,21	3,71	8,91
225	-5,06	301,5	14,01	3,62	-7,09	10,54	5,57	11,45	4,66

Item No.	Item									
41 contd.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	240	-6,20	285,9	19,02	2,88	-6,62	15,28	1,60	17,80	-0,92
	255	-6,92	201,5	15,97	1,88	-4,47	13,38	-1,04	16,03	-3,69
	270	-7,16	134,3	11,21	0,64	-1,10	10,75	4,07	11,98	2,84
	285	-6,92	95,3	7,55	-0,77	2,63	9,41	13,10	8,73	13,78
	300	-6,20	71,9	4,78	-2,24	5,67	8,21	20,53	5,94	22,80
	315	-5,06	45,3	2,10	-3,62	7,09	5,57	28,31	1,39	32,49
	330	-3,57	-3,9	-0,08	-4,74	6,42	1,60	30,41	-5,30	35,71
	345	-1,85	-68,7	0,67	-5,50	3,79	-1,04	23,76	-5,60	28,32
	360	0	-234,3	9,84	-5,77	0	4,07	20,72	1,01	23,78

42	(Maximum) mass of balance weight for reciprocating balance on each main driving wheel at crank radius = [8.(99)]:	kg	11,44
43	Maximum dynamic augment due to [42] at 113 km/h (5,42 Hz with wheel o/d = [2.1.(11)]) = [42] x [2.1.(33)] x $(2 \times \pi \times 5{,}42)^2$ =	kN	5,315

44 [41] cols. (9) & (10) represent the dynamic augment of the main driving wheels due to factors other than reciprocating balance. The vertical component of the force due to the reciprocating balance = [43] x sin (θ- α) for the left wheel and [43] x sin (θ - α + 90^{O}) for the right wheel, see [8.(88)]. The combined dynamic augment is tabulated in [45], for which the columns are as follows:

(1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running.
(2) = [43] x sin (θ- α) = [43] x sin (θ- $2{,}39^{O}$) kN (+ve downwards).
(3) = [43] x sin (θ - α + 90^{O}) = [43] x sin (θ + $87{,}61^{O}$) kN (+ve downwards).
(4) = $R_{l(net)}$ = left main driving wheel dynamic augment = [41]col.(9) + (2) kN (+ve downwards).
(5) = $R_{r(net)}$ = right main driving wheel dynamic augment [41]col.(10) + (3) kN (+ve downwards).
(6) = Main driving axle combined dynamic augment = (4) + (5) kN (+ve downwards).

45	(1)	(2)	(3)	(4)	(5)	(6)
	0	0,22	-5,31	1,23	18,47	19,70
	15	-1,16	-5,19	10,69	15,93	26,62
	30	-2,46	-4,71	18,35	14,04	32,39
	45	-3,60	-3,91	27,20	8,38	35,58
	60	-4,49	-2,85	30,05	0,96	31,01
	75	-5,07	-1,59	22,80	-4,29	18,51
	90	-5,31	-0,22	20,21	-10,40	9,81
	105	-5,19	1,16	18,68	-4,69	13,99
	120	-4,71	2,46	17,01	4,18	21,19
	135	-3,91	3,60	11,65	13,36	25,01
	150	-2,85	4,49	3,74	21,12	24,86
	165	-1,59	5,07	-2,38	20,65	18,27
	180	-0,22	5,31	-8,57	19,03	10,46
	195	1,16	5,19	-2,76	16,67	13,91
	210	2,46	4,71	6,17	13,62	19,79
	225	3,60	3,91	15,05	8,57	23,62
	240	4,49	2,85	22,29	1,93	24,22
	255	5,07	1,59	21,10	-2,10	19,00
	270	5,31	0,22	17,29	3,06	20,35
	285	5,19	-1,16	13,92	12,62	26,54
	300	4,71	-2,46	10,65	20,34	30,99
	315	3,91	-3,60	5,30	28,89	34,19
	330	2,85	-4,49	-2,45	31,22	28,77
	345	1,59	-5,07	-4,01	23,25	19,24
	360	0,22	-5,31	1,23	18,47	19,70

46	**Dynamic augment of coupled axles**. This is calculated by the method of item [44]. Assuming the maximum possible reciprocating balance, the mass of the balance weight for reciprocating balance on each coupled wheel at crank radius = [8.(84)]:	kg	19,92
47	Maximum dynamic augment due to [46] at 113 km/h (5,42 Hz with wheel o/d = [2.1.(11)]) = [46] x [2.1.(33)] x $(2 \times \pi \times 5{,}42)^2$ =	kN	9,255

Item No.	Item				
48	The vertical component of the force due to the reciprocating balance on the coupled wheels = [47] x $\sin(\theta - \alpha)$ for the left wheel and [47] x $\sin(\theta - \alpha + 90^{O})$ for the right wheel, as per item [44]. The combined dynamic augment is tabulated in [49], for which the columns are as follows: (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running: (2) = R_l = left coupled wheel dynamic augment = [47] x $\sin(\theta - \alpha)$ = [47] x $\sin(\theta - 2{,}39^{O})$ kN (+ve downwards). (3) = R_r = right coupled wheel dynamic augment = [47] x $\sin(\theta - \alpha + 90^{O})$ = [47] x $\sin(\theta + 87{,}61^{O})$ kN (+ve downwards). (4) = Total dynamic augment of each coupled axle = (2) + (3) kN (+ve downwards). (5) = **Total dynamic augment of main driving and coupled axles = total unsprung dynamic augment** = [45]col.(6) + (2 x (4)).				
49	(1)	(2)	(3)	(4)	(5)
	0	0,39	-9,25	-8,86	1,98
	15	-2,02	-9,03	-11,05	4,52
	30	-4,29	-8,20	-12,49	7,41
	45	-6,27	-6,81	-13,08	9,42
	60	-7,82	-4,96	-12,78	5,45
	75	-8,83	-2,77	-11,60	-4,69
	90	-9,25	-0,39	-9,64	-9,47
	105	-9,03	2,02	-7,01	-0,03
	120	-8,20	4,29	-3,91	13,37
	135	-6,81	6,27	-0,54	23,93
	150	-4,96	7,82	2,86	30,58
	165	-2,77	8,83	6,06	30,39
	180	-0,39	9,25	8,86	28,18
	195	2,02	9,03	11,05	36,01
	210	4,29	8,20	12,49	44,77
	225	6,27	6,81	13,08	49,78
	240	7,82	4,96	12,78	49,78
	255	8,83	2,77	11,60	42,20
	270	9,25	0,39	9,64	39,63
	285	9,03	-2,02	7,01	40,56
	300	8,20	-4,29	3,91	38,81
	315	6,81	-6,27	0,54	35,27
	330	4,96	-7,82	-2,86	23,05
	345	2,77	-8,83	-6,06	7,12
	360	0,39	-9,25	-8,86	1,98
50	**Change in the weight distribution of the sprung mass due to the wheel rim tractive effort – drawbar pull / engine air resistance couples and the crosshead – slidebar reactions**. The target static weight distribution of the engine is given in Fig. 13.1. This is altered when running due to the effects of the aforementioned couples and reactions. It can also be affected by other loads which act on the sprung mass, for example: (i) eccentric rod forces, (ii) engine friction forces, (iii) horizontal component of the crosshead-slidebar reactions due to the cylinder inclination, (iv) bogie rolling resistance acting horizontally at the bogie pivot, and (v) forces due to unbalanced reciprocating masses. All these are relatively minor and/or have an approximately zero sum over a full coupled wheel revolution, and are ignored in the following analysis.				
51	At any instant the loads acting on the engine's sprung mass, as given in item [50], cause it to accelerate and therefore displace both vertically and rotationally, which changes the spring loads. However apart from the engine air resistance and bogie rolling resistance all these loads vary continuously with coupled wheel crank position, i.e. with coupled wheel rotation. This change is very rapid, undergoing one cycle for every coupled wheel revolution, i.e. 5,42 cycles/second at 113 km/h with wheel o/d = [2.1.(11)]. Furthermore the (target) sprung weight is large (598,6 kN [13.(35)]) and its instantaneous acceleration and displacement are consequently minimal. Therefore in practice there is negligible variation in weight distribution *with crank angle* at a speed such as that for maximum drawbar power, rather there is an approximately constant change which depends on the time-average values of the various forces acting on the sprung mass. This is calculated as follows, referring to Fig. 1.4.3.				

Item No.	Item	Unit	Amount
52	Average wheel rim tractive effort per coupled wheel revolution at maximum drawbar power ≈ average of item [33]col.(3):	kN	75,7
53	Ignoring the small fraction of this force required to move the coupled wheels, the tractive force acting on the sprung mass ≈ [52]:	kN	75,7
54	Height above rail at which [53] acts = coupled wheel radius = [2.1.(11)] ÷ 2:	mm	921
55	For the purpose of these calculations cylinder indicated tractive effort is taken as ≈ [52] ÷ [1.1.(47)]:	kN	78,9
56	Target total engine mass = Σ [13.(4)]:	ton	80
57	From equations [1.1.(24)] & [1.1.(26)], the rolling resistance of the engine (less tender) at 113 km/h ([1.3.(2)]) ≈ $\{45 + (0{,}24 \times [1.3.(2)]) + (0{,}0036 \times [1.3.(2)]^2) - (0{,}00045 \times [1.3.(2)]^2)\} \times [56]$:	kN	9,0
58	At indicated t.e. = [55] drawbar pull between engine and tender ≈ [55] – [57]:	kN	69,9
59	Air resistance of sprung mass at speed [1.3.(2)] ≈ [53] – [58]:	kN	5,8
60	[58] acts at the engine-tender drawbar, of height above rail level, from Ref. [3] and referred to coupled wheel diameter [2.1.(11)]:	mm	1 035
61	[59] acts at the vertical centre of gravity of the sprung mass, which is estimated using Refs. [3] & [4] as ≈	mm	2 400
62	The average value per coupled wheel revolution of the combined left and right side crosshead-slidebar reactions is found by the following table, for which the columns are as follows: (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running. (2) = left connecting rod deviation (modulus value), φ degrees, from the straight line joining the cylinder centre line to the driving axle centre line and found by the equation given in Fig. 2.2.3. item (1). (3) = thrust in left connecting rod, kN, = [3.(287)] col. (14). (4) = right connecting rod deviation (modulus value), φ degrees. (5) = thrust in right connecting rod, kN. (6) = (3) x sin (2), kN. (7) = (5) x sin (4), kN. (8) = combined left and right side crosshead-slidebar reactions = (6) + (7), kN.		

63	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	0	0	97,0	7,16	129,5	0	16,14	16,14
	15	1,85	170,8	6,92	128,4	5,51	15,47	20,98
	30	3,57	191,3	6,20	132,6	11,91	14,32	26,23
	45	5,06	213,8	5,06	117,9	18,86	10,40	29,26
	60	6,20	202,5	3,57	78,4	21,87	4,88	26,75
	75	6,92	149,1	1,85	20,0	17,96	0,65	18,61
	90	7,16	129,5	0	-128,1	16,14	0	16,14
	105	6,92	128,4	1,85	197,3	15,47	6,37	21,84
	120	6,20	132,6	3,57	206,5	14,32	12,86	27,18
	135	5,06	117,9	5,06	216,2	10,40	19,07	29,47
	150	3,57	78,4	6,20	218,7	4,88	23,62	28,50
	165	1,85	20,0	6,92	157,9	0,65	19,02	19,67
	180	0	-128,1	7,16	120,1	0	14,97	14,97
	195	1,85	197,3	6,92	114,4	6,37	13,78	20,15
	210	3,57	206,5	6,20	126,0	12,86	13,61	26,47
	225	5,06	216,2	5,06	132,0	19,07	11,64	30,71
	240	6,20	218,7	3,57	109,2	23,62	6,80	30,42
	255	6,92	157,9	1,85	62,3	19,02	2,01	21,03
	270	7,16	120,1	0	-97,0	14,97	0	14,97
	285	6,92	114,4	1,85	170,8	13,78	5,51	19,29
	300	6,20	126,0	3,57	191,3	13,61	11,91	25,52
	315	5,06	132,0	5,06	213,8	11,64	18,86	30,50
	330	3,57	109,2	6,20	202,5	6,80	21,87	28,67
	345	1,85	62,3	6,92	149,1	2,01	17,96	19,97
	360	0	-97,0	7,16	129,5	0	16,14	16,14

Item No.	Item	Unit	Amount
64	The average value of [63]col.(8) per coupled wheel revolution =	kN	23,48

Item No.	Item	Unit	Amount
65	Vertical component of [64] = [64] x cos [2.1.(347)]: (The horizontal component [64] x sin [2.1.(347)] = 0,98 kN is small and is ignored.)	kN	23,46 say 23,5
66	[65] is taken to act at the centre of the slidebars (= the centre of the crosshead at mid-stroke), longitudinal distance from the bogie centre line ≈ [2.2.(197)] x cos [2.1.(347)]:	mm	1 553
67	Taking moments about X, Fig. 1.4.3.: ([13.(35)] x [13.(36)]) + ([59] x ([61] – [54])) + ([58] x ([60] – [54])) = ([65] x [66]) + L_C x [13.(37)] where L_C = spring-borne load on driving & coupled wheels.		
68	Substituting numerical data into equation [67], L_C =	kN	442,3
69	Load on bogie = L_B = [13.(35)] – [65] – [68]:	kN	132,8
70	Static value of L_C = [13.(29)]:	kN	446,4
71	Static value of L_B = [13.(34)]:	kN	152,2
72	Average loss of spring-borne load on driving & coupled wheels over a full coupled wheel revolution at maximum drawbar power = [70] – [68]:	kN	4,1
73	[72] ÷ [70] =	%	0,9
74	Average loss of load on bogie over a full coupled wheel revolution at maximum drawbar power = [71] – [69]:	kN	19,4
75	[74] ÷ [71] =	%	12,7
76	[72] & [73] are negligible, but [74] & [75] must be taken into consideration when calculating the bogie lateral guiding force.		
77	The instantaneous coefficient of adhesion required to prevent slipping at the chosen conditions is calculated in the following table, for which the columns are as follows: (1) = crank angle, θ degrees, of the left crank from f.d.c. for forward running. (2) = static adhesive weight = 588,6 kN ([26]). (3) = unsprung dynamic augment = [49]col.(5), kN. (4) = sprung dynamic augment = – 4,1 kN ([72]). (5) = total dynamic adhesive ‘weight’ = (2) + (3) + (4), kN. (6) = wheel rim tractive effort = [33]col.(3), kN. (7) = instantaneous coefficient of adhesion required to transmit the tractive effort = [6] ÷ [5].		

Item No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
78	0	588,6	1,98	– 4,1	586,48	51,9	0,09
	15	588,6	4,52	– 4,1	589,02	68,1	0,12
	30	588,6	7,41	– 4,1	591,91	85,9	0,15
	45	588,6	9,42	– 4,1	593,92	96,8	0,16
	60	588,6	5,45	– 4,1	589,95	89,0	0,15
	75	588,6	– 4,69	– 4,1	579,81	61,4	0,11
	90	588,6	– 9,47	– 4,1	575,03	51,9	0,09
	105	588,6	– 0,03	– 4,1	584,47	66,2	0,11
	120	588,6	13,37	– 4,1	597,87	80,4	0,13
	135	588,6	23,93	– 4,1	608,43	86,7	0,14
	150	588,6	30,58	– 4,1	615,08	85,4	0,14
	165	588,6	30,39	– 4,1	614,89	61,0	0,10
	180	588,6	28,18	– 4,1	612,68	48,2	0,08
	195	588,6	36,01	– 4,1	620,51	63,9	0,10
	210	588,6	44,77	– 4,1	629,27	83,7	0,13
	225	588,6	49,78	– 4,1	634,28	97,0	0,15
	240	588,6	49,78	– 4,1	634,28	95,6	0,15
	255	588,6	42,20	– 4,1	626,70	66,4	0,11
	270	588,6	39,63	– 4,1	624,13	48,2	0,08
	285	588,6	40,56	– 4,1	625,06	65,7	0,11
	300	588,6	38,81	– 4,1	623,31	89,3	0,14
	315	588,6	35,27	– 4,1	619,77	107,1	0,17
	330	588,6	23,05	– 4,1	607,55	99,2	0,16
	345	588,6	7,12	– 4,1	591,62	66,8	0,11
	360	588,6	1,98	– 4,1	586,48	51,9	0,09

Item No.	Item
79	[78]col.(7) is plotted against crank angle θ in Fig. 1.4.4.: it is seen that the required instantaneous coefficient of adhesion is never greater than that which should be available on dry or wet rails at a speed of 113 km/h (see items [34], [35], [1.1.(38)-(45)] and Fig. 1.1.2.). However it would probably rise above that for very poor adhesion conditions, such as wet leaves, drizzle, or ice/snow, for which sand will be required, see item [38].

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4. British Railways drawing No. SL/DN/P/120, Class 5MT 4-6-0, General Arrangement, End Views & Cross Sections.